Trigonometric Identities

There are several identities (formulas) involving trigonometric functions that are true for every angle.

Some are crazy complicated (and that's what precalc is for), but there are a few which are very simple and can be used to simplify complicated expressions and equations.

Reciprocal Identities

$\frac{1}{sin\theta} = csc\theta$ $\qquad$ $\frac{1}{csc\theta} = sin\theta$

$\frac{1}{cos\theta} = sec\theta$ $\qquad$ $\frac{1}{sec\theta} = cos\theta$

$\frac{1}{tan\theta} = cot\theta$ $\qquad$ $\frac{1}{cot\theta} = tan\theta$

Proof:

$$\frac{sin\theta}{cos\theta} = \frac{\frac{opp}{hyp}}{\frac{adj}{hyp}} = \frac{opp}{hyp} \times \frac{hyp}{adj} = \frac{opp}{adj} = tan\theta$$

$$\frac{cos\theta}{sin\theta} = \frac{\frac{adj}{hyp}}{\frac{opp}{hyp}} = \frac{adj}{hyp} \times \frac{hyp}{opp} = \frac{adj}{opp} = cot\theta$$

Quotient Identities:

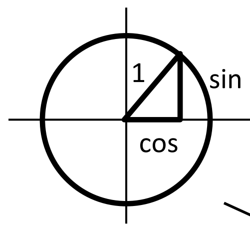
$tan\theta = \frac{sin\theta}{cos\theta}$ $\qquad$ $cot\theta = \frac{cos\theta}{sin\theta}$

Pythagorean Identities:

$sin^2\theta + cos^2\theta = 1$

$tan^2\theta + 1 = sec^2\theta$

$1 + cot^2\theta = csc^2\theta$



In the unit circle,

The Pythagorean Theorem yields the first Pythagorean identity.

Dividing each term by $cos^2\theta$: $\frac{sin^2\theta}{cos^2\theta} + \frac{cos^2\theta}{cos^2\theta} = \frac{1}{cos^2\theta}$ and using the quotient identities yields the second one.

Dividing each term by $sin^2\theta$ yields the third one.

So how do we use these?

Proving identities and simplifying expressions.

Ex: $\frac{1-cos^2\theta}{sin^2\theta} = 1 \rightarrow \frac{sin^2\theta}{sin^2\theta} = 1$

Because: $\frac{1-cos^2\theta}{sin^2\theta} = \frac{(sin^2\theta+cos^2\theta)-cos^2\theta}{sin^2\theta} = \frac{sin^2\theta+\cancel{cos^2\theta}-\cancel{cos^2\theta}}{sin^2\theta} = \frac{sin^2\theta}{sin^2\theta} = 1$

Ex: $cos\theta + sin\theta \cdot tan\theta = cos\theta + sin\theta \cdot \frac{sin\theta}{cos\theta} = cos\theta + \frac{sin^2\theta}{cos\theta}$

Multiply the first term by 1 (in the form of $\frac{cos\theta}{cos\theta}$) to get a common denominator.

$$\frac{cos\theta}{cos\theta} \cdot \frac{cos\theta}{1} + \frac{sin^2\theta}{cos\theta} = \frac{cos^2\theta}{cos\theta} + \frac{sin^2\theta}{cos\theta} = \frac{(cos^2\theta + sin^2\theta)}{cos\theta} = \frac{1}{cos\theta} = sec\theta$$